

Under the Hood of the MJS College Football Standings

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1 What are we trying to accomplish?

In most sports, team standings are determined by win-loss records. But college football doesn't work that way. There are hundreds of college football teams, with each team playing only about a dozen games a year. Some teams have a tough schedule filled with big-budget opponents. Others play mostly small-budget teams with players that actually attend class. Standings based solely on win-loss percentage don't work – there is simply too much variation in the toughness of team schedules.

So college football has its polls. A panel of sportwriters (AP) or coaches (USA Today / ESPN) votes each week on what the standings should be. This isn't very satisfying. Panels of judges may have a place in figure skating, but in football? C'mon!

What college football needs are standings that rank teams according to who has the best record, where “best record” is determined by simple, objective rules. That would eliminate the dependence on polls and make for standings that everybody can agree are correct according to the rules.

Objective standings that rank teams according to their records should adhere to a few basic principles:

- **A win is a win.** Win by 1 point, win by 50. Win impressively, or win by dumb luck. It doesn't matter. *A win is a win.*
- **All games are equally important.** The first game of the season counts as much as the last. No forgiveness for losing early, no added penalty for losing late.
- **Nothing else matters.** Home field advantage, injuries, team statistics, player statistics, weather, bad officiating, etc. – none of it matters to the standings.
- **Simple and published.** The rules for computing standings should be simple and published, so that anybody who wishes to compute or verify them can do so. This is not the case with the mysterious and complicated “computer rankings” used in the BCS standings. Nobody besides the BCS and its affiliated experts knows how the computer rankings work. It's no surprise there is not widespread acceptance of their validity.

2 Description of MJS Standings

The MJS Standings compute a *rating* for each Division 1A team. The teams are *ranked* in order of rating, with the highest-rated team ranked number one. The rating is determined by combining a team's *winning percentage* with its *strength of schedule*. The details require a little bit of math – hang in there, it's not much.

The winning percentage (WP) of a team is the well-known formula:

$$\text{WP} = \text{wins} / \text{games} .$$

A team that plays 11 games and wins 8 of them has a WP of $8/11 = 0.72727$.

The strength of schedule (SOS) of a team is the average value of its opponents' ratings. That is:

$$\text{SOS} = (\text{sum of opponents' ratings}) / \text{games} .$$

The reasoning behind this is simple: the strength of an opponent is indicated by its rating. So it makes sense to define a team's strength of schedule as the average value of the ratings of its opponents.

A team's rating is the sum of its WP and its SOS. That is:

$$\text{rating} = \text{WP} + \text{SOS} .$$

By this formula, if two teams have identical WPs, the team with the higher SOS has the higher rating. Or if two teams have identical SOS's, then the team with the higher WP has the higher rating. In general, a team with a strong SOS gets a boost in its rating relative to a team with a weak SOS; that boost may be enough to allow it to surpass a team with a higher WP.

That's just about the whole story, but there are a couple of technical tweaks.

The first tweak is that the actual formula for SOS is a little different than the one shown above. It is actually:

$$\text{SOS} = 0.7 * [(\text{sum of opponent's ratings}) / \text{games} - 0.5] .$$

The value 0.5 is used because it makes the average value of SOS zero. This is convenient, because then a positive SOS means a tougher-than-average schedule, while a negative SOS means a weaker-than-average schedule.

The value of 0.7 is used to adjust the relative importance of SOS in a team's rating. This value was determined by trial and error. The ratings were computed for past seasons using a range of values, and resulting rankings were observed. The value of 0.7 was selected because it produces rankings that are a good match to the polls of past seasons.

The second tweak is for handling games between a team in Division 1A and a team not in Division 1A. All such non-Division-1A opponents are lumped together and treated as a single team, "X". So, in the case of the 2005 season, the ratings are computed for 120 "teams": the 119 Division-1A teams, and the non-Division-1A team X. This may seem a little imprecise, but as a practical matter it works out well – team X always ends up with

a very low rating, very close to the worst-rated Division-1A team. Note that games between two non-Division-1A teams are ignored – only games that include a Division-1A team are considered.

3 Example: 2004 MJS standings

Let's see how all this works out by looking at an actual example, the final MJS Standings for 2004. This is an interesting example because the national championship picture was controversial that year, with a handful of teams ending up undefeated both before and after the bowl games.

The standings are shown in Appendix 1, with backup data in Appendix 2 showing each team's WP, SOS, and rating. The top two teams are Southern_cal and Auburn, each with identical records of 13-0. Southern_cal is ranked number one because its rating (1.06187) is higher than Auburn's (1.02932). There is a third undefeated team, Utah, but it is only ranked number five because its rating is lower than some teams with losses.

We can verify Southern_cal's rating easily. First, we add up the ratings of Southern_cal's opponents:

3	oklahoma	1.00379
9	california	0.87230
11	arizona_st	0.84262
14	virginia_tech	0.80687
27	oregon_st	0.67388
42	notre_dame	0.59054
56	ucla	0.53017
58	brigham_young	0.52235
65	washington_st	0.48601
74	stanford	0.41587
77	colorado_st	0.40300
84	arizona	0.36068
112	washington	<u>0.14087</u>
		7.64895

Since Southern_cal has 13 opponents, the average rating of its opponents is this sum divided by 13. We compute Southern_cal's SOS like this:

$$\text{SOS} = 0.7 * (7.64895/13 - 0.5) = 0.06187.$$

Southern_cal's rating is the sum of its WP and its SOS:

$$\text{Rating} = 1.00000 + 0.06187 = 1.06187.$$

We can verify Auburn's rating in the same way. We add up the ratings of Auburn's opponents:

10	georgia	0.87146
13	tennessee	0.82332
13	tennessee	0.82332
14	virginia_tech	0.80687
16	lsu	0.79075
50	louisiana_tech	0.55484
54	arkansas	0.53534
60	alabama	0.50495
78	mississippi	0.40125
83	ul_monroe	0.36074
97	mississippi_st	0.26212
103	kentucky	0.23066
x	_not_div_1a	<u>0.07880</u>
		7.04442

Note that Tennessee is included twice in the list of opponents, because Auburn played Tennessee twice. Note also that team X is included, once, because Auburn played one non-Division-1A team. Auburn's SOS is computed as:

$$\text{SOS} = 0.7 * (7.04442/13 - 0.5) = 0.02931.$$

Auburn's rating is computed as WP plus SOS:

$$\text{Rating} = 1.00000 + 0.02931 = 1.02931.$$

There's a slight difference between this value and the one listed in the appendix (1.02932). This is due to rounding of the listed values to five decimal places.

Both Southern_cal and Auburn have an SOS that is positive. In other words, their schedule was tougher than average. The third undefeated team, Utah, has an SOS that is negative (-0.01822), meaning its schedule was weaker than average. It is Utah's weak schedule that causes it to rank below the other undefeated teams – indeed, it even ranks below several teams that were not undefeated but had tougher schedules.

The team with the toughest schedule is Texas_a&m (+0.13992). Its SOS effectively boosts its record by $12 * 0.13992 = 1.68$ wins. That is, its actual record of 7 wins, 5 losses against its tough schedule produces the same rating as a record of 8.68 wins, 3.32 losses would produce against an average schedule.

The weakest schedule belongs to Navy (-0.13113). Its SOS degrades its record by $12 * 0.13113 = 1.57$ losses, meaning its actual record of 10 wins, 2 losses against its weak schedule gives it the same rating as 8.43 wins, 3.57 losses against an average schedule.

Team X is worth a look. Its record of 6 wins, 50 losses, means there were 56 games between a Division-1A team and a non-Division-1A opponent, with 50 of those games being won by the Division-1A team. Team X's SOS (-0.02834) is below average. In other words, not many strong Division-1A teams played non-Division-1A opponents. Team X's rating is 0.07880, which would give it a rank of 115 if it were included in the standings.

4 Computing the MJS Standings

The example above shows how to verify a team's rating in the MJS Standings. To do this, it isn't necessary to know how to compute the ratings from scratch. It is only necessary to check that the team's rating satisfies the formulas:

$$\begin{aligned} \text{SOS} &= 0.7 * [(\text{sum of opponents' rating})/\text{games} - 0.5] , \\ \text{rating} &= \text{WP} + \text{SOS}. \end{aligned}$$

An important mathematical fact about the MJS Standings is this: there is only one correct answer. That is, there is only one set of ratings that will satisfy the formulas for every team. If I hand you the computed ratings, and you verify that the rating for each team is correct according to the formulas, then you know that the ratings I handed you are the official MJS Standings. I can't fudge it – there is only one correct answer, and you can check it.

So to a large degree, there really is no need to worry about how to compute the MJS Standings. Anybody can verify that the published ratings are correct. There is no mystery, no secret sauce.

That said, if one wishes to compute the MJS Standings for oneself, it's a straightforward computer programming task (Warning: geek-speak follows.) The ratings are computed by solving a system of simultaneous linear equations. The number of equations is equal to the number of Division-1A teams plus one, the "plus one" accounting for the non-Division-1A team X. The equations are well-conditioned, so no elaborate numerical tricks are needed. (Relax: geek-speak done.) If this lingo is familiar to you, then you're all set – fire up the machine, crank out some code, and compute the standings for yourself. If not, don't worry about it – just look at the published MJS Standings and verify whatever part of it you wish.

44	cincinnati	7-5	36	47	57	69	72	86	111	/	8	21	52	55	110	
45	clemson	6-5	12	51	62	63	81	104	/	17	20	22	33	101		
46	minnesota	7-5	37	53	60	77	80	94	x	/	7	15	19	71	98	
47	southern_miss	7-5	52	59	64	75	86	92	111	/	9	36	44	60	69	
48	hawaii	8-5	50	52	53	71	89	91	108	113	/	6	25	31	95	x
49	wyoming	7-5	56	67	78	82	83	106	x	/	5	20	38	58	77	
50	louisiana_tech	6-6	25	89	91	93	95	99	/	2	6	12	13	31	48	
51	so_carolina	6-5	54	60	61	86	103	109	/	10	13	39	45	78		
52	alabama_birm	7-5	36	44	69	85	92	97	110	/	17	47	48	75	86	
53	northwestern	6-6	21	35	70	80	94	98	/	11	15	19	46	48	69	
54	arkansas	5-6	60	78	79	83	97	/	2	4	10	16	39	51		
55	syracuse	6-6	24	30	41	44	88	115	/	17	22	33	34	35	107	
56	ucla	6-6	66	74	82	84	94	112	/	1	9	11	28	49	65	
57	miami_ohio	8-5	37	73	90	96	115	116	117	x	/	15	37	40	44	72
58	brigham_young	5-6	42	49	67	77	82	/	1	5	6	38	74	106		
59	north_texas	7-5	79	83	87	99	104	105	108	/	4	26	47	85	x	
60	alabama	6-6	47	78	97	103	104	x	/	2	13	16	46	51	54	
61	troy_st	7-5	68	72	87	99	104	108	x	/	16	32	51	79	105	
62	maryland	5-6	17	32	81	101	107	/	14	22	33	34	45	63		
63	no_carolina_st	5-6	14	62	81	111	x	/	12	17	21	33	43	45		
64	nebraska	5-6	30	68	70	85	x	/	3	18	26	40	47	76		
65	washington_st	5-6	38	56	84	108	112	/	1	11	26	27	66	74		
66	oregon	5-6	65	74	84	108	112	/	3	9	11	27	56	98		
67	air_force	5-6	38	77	106	110	x	/	5	9	23	49	58	82		
68	missouri	5-6	26	40	85	105	114	/	4	28	61	64	70	76		
69	texas_christian	5-6	47	53	92	93	110	/	8	18	44	52	75	86		
70	kansas	4-7	37	68	76	91	/	3	4	18	26	40	53	64		
71	michigan_st	5-7	19	46	94	98	102	/	7	15	21	42	48	80	88	
72	marshall	6-6	57	90	96	115	116	117	/	10	21	29	44	61	73	
73	akron	6-5	72	90	96	114	115	117	/	22	32	57	80	87		
74	stanford	4-7	58	65	112	113	/	1	9	11	27	42	56	66		
75	tulane	5-6	23	52	69	110	x	/	8	36	47	92	97	111		
76	kansas_st	4-7	64	68	99	x	/	3	18	20	25	26	40	70		
77	colorado_st	4-7	49	82	106	x	/	1	5	26	38	46	58	67		
78	mississippi	4-7	51	97	105	109	/	2	13	16	36	49	54	60		
79	new_mexico_st	5-6	61	87	99	x	x	/	9	31	38	54	59	104		
80	penn_st	4-7	71	73	98	117	/	7	19	21	24	35	46	53		
81	wake_forest	4-7	24	101	111	x	/	12	14	17	43	45	62	63		
82	san_diego_st	4-7	67	89	106	x	/	5	15	38	49	56	58	77		
83	ul_monroe	5-6	99	104	108	x	x	/	2	49	54	59	87	105		
84	arizona	3-8	11	112	x	/	1	5	9	19	27	56	65	66		
85	baylor	3-8	20	59	x	/	3	4	18	28	40	52	64	68		
86	so_florida	4-7	52	69	111	x	/	8	30	36	44	47	51	110		
87	mid_tennessee_st	5-6	73	83	104	105	108	/	39	59	61	79	99	x		
88	rutgers	4-7	71	90	107	109	/	23	24	30	34	41	55	x		
89	nevada_reno	5-7	91	95	113	115	x	/	6	25	48	50	82	93	106	
90	kent_st	5-6	96	100	115	117	x	/	7	57	72	73	88	102		
91	tulsa	4-8	31	95	113	x	/	6	23	28	48	50	70	89	93	
92	houston	3-8	75	110	111	/	3	8	12	36	47	52	69	95		
93	smu	3-8	89	91	113	/	6	18	25	28	31	50	69	95		
94	illinois	3-8	98	116	x	/	7	15	19	35	46	53	56	71		
95	rice	3-8	48	92	93	/	4	23	25	31	50	89	91	113		
96	ohio	4-7	103	115	117	x	/	29	30	37	57	72	73	90		
97	mississippi_st	3-8	39	75	103	/	2	16	52	54	60	78	109	x		
98	indiana	3-8	46	66	102	/	15	21	35	53	71	80	94	103		
99	ul_lafayette	4-7	87	105	x	x	/	50	59	61	76	79	83	108		
100	e_michigan	4-7	102	114	115	116	/	29	32	37	39	90	108	x		
101	duke	2-9	45	x	/	14	17	22	23	33	41	43	62	81		
102	cent_michigan	4-7	90	114	116	x	/	29	32	37	71	98	100	115		
103	kentucky	2-9	98	109	/	2	8	10	13	39	51	60	96	97		
104	utah_st	3-8	79	106	108	/	5	45	59	60	61	83	87	105		
105	arkansas_st	3-8	61	83	104	/	16	36	59	68	78	87	99	108		
106	nev_las_vegas	2-9	58	89	/	5	13	19	38	49	67	77	82	104		

107	temple	2-9	55	x /	22	24	29	30	34	37	41	62	88	
108	idaho	3-9	99	100	105 /	6	48	59	61	65	66	83	87	104
109	vanderbilt	2-9	97	x /	10	13	16	23	39	51	78	88	103	
110	army	2-9	44	86 /	8	23	41	52	67	69	75	92	111	
111	e_carolina	2-9	75	110 /	8	34	36	44	47	63	81	86	92	
112	washington	1-10	113	/	1	9	25	27	42	56	65	66	74	84
113	san_jose_st	2-9	95	x /	6	25	31	48	74	89	91	93	112	
114	ball_st	2-9	116	117 /	24	29	32	35	37	68	73	100	102	
115	buffalo	2-9	102	117 /	41	55	57	72	73	89	90	96	100	
116	western_mich	1-10	x	/	14	29	32	37	57	72	94	100	102	114
117	cent_florida	0-11	/	19	32	34	57	72	73	80	90	96	114	115

6 Appendix 2: Backup data to 2004 MJS Standings

	Team	W-L	PCT	SOS	Rating
1	southern_cal	13-0	1.00000	0.06187	1.06187
2	auburn	13-0	1.00000	0.02932	1.02932
3	oklahoma	12-1	0.92308	0.08071	1.00379
4	texas	11-1	0.91667	0.06937	0.98604
5	utah	12-0	1.00000	-0.01822	0.98178
6	boise_st	11-1	0.91667	-0.01139	0.90528
7	iowa	10-2	0.83333	0.06559	0.89892
8	louisville	11-1	0.91667	-0.02181	0.89485
9	california	10-2	0.83333	0.03897	0.87230
10	georgia	10-2	0.83333	0.03813	0.87146
11	arizona_st	9-3	0.75000	0.09262	0.84262
12	miami_fla	9-3	0.75000	0.07846	0.82846
13	tennessee	10-3	0.76923	0.05409	0.82332
14	virginia_tech	10-3	0.76923	0.03764	0.80687
15	michigan	9-3	0.75000	0.04658	0.79658
16	lsu	9-3	0.75000	0.04075	0.79075
17	florida_st	9-3	0.75000	0.03711	0.78711
18	texas_tech	8-4	0.66667	0.07549	0.74216
19	wisconsin	9-3	0.75000	-0.00871	0.74129
20	texas_a&m	7-5	0.58333	0.13992	0.72325
21	ohio_st	8-4	0.66667	0.05054	0.71721
22	virginia	8-4	0.66667	0.04849	0.71516
23	navy	10-2	0.83333	-0.13113	0.70220
24	boston_college	9-3	0.75000	-0.05166	0.69834
25	fresno_st	9-3	0.75000	-0.05636	0.69364
26	colorado	8-5	0.61538	0.07122	0.68661
27	oregon_st	7-5	0.58333	0.09054	0.67388
28	oklahoma_st	7-5	0.58333	0.08148	0.66481
29	bowling_green	9-3	0.75000	-0.08696	0.66304
30	pittsburgh	8-4	0.66667	-0.01364	0.65303
31	texas_el_paso	8-4	0.66667	-0.01590	0.65076
32	northern_ill	9-3	0.75000	-0.11832	0.63168
33	georgia_tech	7-5	0.58333	0.04820	0.63153
34	west_virginia	8-4	0.66667	-0.03972	0.62695
35	purdue	7-5	0.58333	0.04111	0.62444
36	memphis	8-4	0.66667	-0.04651	0.62016
37	toledo	9-4	0.69231	-0.07569	0.61662
38	new_mexico	7-5	0.58333	0.02954	0.61288
39	florida	7-5	0.58333	0.02689	0.61022
40	iowa_st	7-5	0.58333	0.02095	0.60428
41	connecticut	8-4	0.66667	-0.06513	0.60153
42	notre_dame	6-6	0.50000	0.09054	0.59054
43	no_carolina	6-6	0.50000	0.08934	0.58934
44	cincinnati	7-5	0.58333	0.00048	0.58381
45	clemson	6-5	0.54545	0.03615	0.58160
46	minnesota	7-5	0.58333	-0.00340	0.57993
47	southern_miss	7-5	0.58333	-0.01092	0.57242
48	hawaii	8-5	0.61538	-0.04595	0.56944
49	wyoming	7-5	0.58333	-0.02066	0.56267
50	louisiana_tech	6-6	0.50000	0.05484	0.55484
51	so_carolina	6-5	0.54545	0.00655	0.55200
52	alabama_birm	7-5	0.58333	-0.03252	0.55082
53	northwestern	6-6	0.50000	0.04031	0.54031
54	arkansas	5-6	0.45455	0.08080	0.53534
55	syracuse	6-6	0.50000	0.03212	0.53212
56	ucla	6-6	0.50000	0.03017	0.53017
57	miami_ohio	8-5	0.61538	-0.09044	0.52494
58	brigham_young	5-6	0.45455	0.06780	0.52235
59	north_texas	7-5	0.58333	-0.07725	0.50609

60	alabama	6-6	0.50000	0.00495	0.50495
61	troy_st	7-5	0.58333	-0.08279	0.50055
62	maryland	5-6	0.45455	0.03844	0.49298
63	no_carolina_st	5-6	0.45455	0.03633	0.49087
64	nebraska	5-6	0.45455	0.03211	0.48665
65	washington_st	5-6	0.45455	0.03146	0.48601
66	oregon	5-6	0.45455	0.01798	0.47252
67	air_force	5-6	0.45455	-0.00040	0.45414
68	missouri	5-6	0.45455	-0.00122	0.45333
69	texas_christian	5-6	0.45455	-0.00555	0.44899
70	kansas	4-7	0.36364	0.08496	0.44860
71	michigan_st	5-7	0.41667	0.02262	0.43929
72	marshall	6-6	0.50000	-0.06148	0.43852
73	akron	6-5	0.54545	-0.10948	0.43597
74	stanford	4-7	0.36364	0.05223	0.41587
75	tulane	5-6	0.45455	-0.04526	0.40929
76	kansas_st	4-7	0.36364	0.04243	0.40607
77	colorado_st	4-7	0.36364	0.03936	0.40300
78	mississippi	4-7	0.36364	0.03762	0.40125
79	new_mexico_st	5-6	0.45455	-0.05375	0.40080
80	penn_st	4-7	0.36364	0.02400	0.38764
81	wake_forest	4-7	0.36364	0.01703	0.38067
82	san_diego_st	4-7	0.36364	-0.00104	0.36260
83	ul_monroe	5-6	0.45455	-0.09380	0.36074
84	arizona	3-8	0.27273	0.08795	0.36068
85	baylor	3-8	0.27273	0.08273	0.35546
86	so_florida	4-7	0.36364	-0.01200	0.35163
87	mid_tennessee_st	5-6	0.45455	-0.10927	0.34527
88	rutgers	4-7	0.36364	-0.02770	0.33593
89	nevada_reno	5-7	0.41667	-0.09023	0.32643
90	kent_st	5-6	0.45455	-0.12862	0.32593
91	tulsa	4-8	0.33333	-0.02407	0.30926
92	houston	3-8	0.27273	0.02918	0.30190
93	smu	3-8	0.27273	0.01254	0.28526
94	illinois	3-8	0.27273	0.00163	0.27436
95	rice	3-8	0.27273	0.00047	0.27320
96	ohio	4-7	0.36364	-0.09626	0.26738
97	mississippi_st	3-8	0.27273	-0.01061	0.26212
98	indiana	3-8	0.27273	-0.01279	0.25994
99	ul_lafayette	4-7	0.36364	-0.11792	0.24572
100	e_michigan	4-7	0.36364	-0.12115	0.24249
101	duke	2-9	0.18182	0.05522	0.23704
102	cent_michigan	4-7	0.36364	-0.12757	0.23606
103	kentucky	2-9	0.18182	0.04884	0.23066
104	utah_st	3-8	0.27273	-0.04380	0.22892
105	arkansas_st	3-8	0.27273	-0.05414	0.21859
106	nev_las_vegas	2-9	0.18182	0.03305	0.21487
107	temple	2-9	0.18182	0.03274	0.21456
108	idaho	3-9	0.25000	-0.05357	0.19643
109	vanderbilt	2-9	0.18182	0.01010	0.19192
110	army	2-9	0.18182	-0.00156	0.18026
111	e_carolina	2-9	0.18182	-0.00555	0.17627
112	washington	1-10	0.09091	0.04996	0.14087
113	san_jose_st	2-9	0.18182	-0.05417	0.12765
114	ball_st	2-9	0.18182	-0.05858	0.12324
115	buffalo	2-9	0.18182	-0.10354	0.07828
116	western_mich	1-10	0.09091	-0.05494	0.03597
117	cent_florida	0-11	0.00000	-0.05843	-0.05843
x	_not_div_la	6-50	0.10714	-0.02834	0.07880