



Tontine Pensions

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Tontines

- Investment vehicles that combine features of an annuity & a lottery
- Investors pool their money
 - Each year they are alive, members get investment income
 - As members die, their shares are forfeited to the surviving members (“mortality gains”)
- Unless the fund is divided earlier, the entire fund goes to the last survivor



Example 1

- On the TV show, M*A*S*H, Colonel Sherman T. Potter, as the last survivor of his World War I unit, got to open the bottle of cognac that he and his buddies got in France – and share it with his Korean War compatriots





Example 2

Steve and Mark are camping when a bear suddenly comes out and growls.

Steve starts putting on his tennis shoes.

Mark says, “What are you doing? You can’t outrun a bear!”

Steve says, “I don’t have to outrun the bear—I just have to outrun you!”*

* Boys’ Life, *You can’t outrun a bear*, <http://boyslife.org/jokes/6953/you-cant-outrun-a-bear/> (last visited Mar. 5, 2015).



Ex. 3: A Simple Tontine Fund

- Imagine a fund with 4 investors of different ages & each contributes \$1000
- When one dies, each survivor gets \$333.33
$$\$333.33 = \$1000 \div 3$$
- Unfortunately, this simple approach can be unfair
 - E.g., because it favors younger investors
 - who are likely to live longer & so get more distributions than older investors



Example 4: What Would Be Fair?

- Imagine that our 4 investors are ages
 - 65, 70, 75 & 80
- Let's make it a fair bet for everyone
- Start with death probabilities (q_i)
 - The probability of dying within the next year
 - These come from a life expectancy table
 - E.g., a 65-year-old
 - has a *life expectancy* ~ 19 years
 - has a 1.3% chance of dying before age 66 (*death probability*)



A Fair Transfer Plan

- Use these death probabilities (q_i) to derive “fair-transfer-plan weights” (w_i)
- Use these fair-transfer-plan weights (w_i) to divide the accounts of those who die
- Result is a “fair transfer plan” (FTP)



Example 4: Unisex Table

<i>Member</i>	<i>Age</i>	<i>Life Expectancy (years)</i>	<i>Death Probability</i>	<i>Fair-Transfer-Plan Weight</i>
1	65	18.88	0.013181	0.053815
2	70	15.22	0.020314	0.086183
3	75	11.89	0.032111	0.146795
4	80	8.95	0.051906	0.713207

- E.g., 65-year-old member 1:
 - has the longest *life expectancy* ~ 19 years
 - has the lowest *death probability* ~ 1.3%
 - therefore, has the smallest *fair-transfer-plan weight* ~ 0.05



Example 4: A Fair Result

- For example, suppose member 4 (**the 80-year-old**) is the first to die
- Her \$1000 would be distributed as follows:
 - 65-year-old member 1 gets \$ 187.64
 - 70-year-old member 2 gets \$ 300.51
 - 75-year-old member 3 gets \$ 511.85
 - 80-year-old member 4 loses \$ 1000.00



Example 4: The Math

- If member j dies, each surviving member i would get a portion of j 's contribution equal to $w_i/(1 - w_j)$, for $i \neq j$.
- So when member 4 (the 80-year-old) dies, member 1 (the 65-year-old) gets

$$\begin{aligned} \$187.64 &= \$1000 \times w_1/(1 - w_4) \\ &= \$1000 \times 0.053815/(1 - 0.713207) \end{aligned}$$



Example 4: The Logic

- Younger investors get less now, but they should live longer & collect more payments
 - In short, a tontine fund can fairly accommodate members of different ages
 - It's a fair bet
- Tontine funds can also be fair to members with different contribution levels
- Tontine funds could also be **perpetual**, with new investors coming in all the time

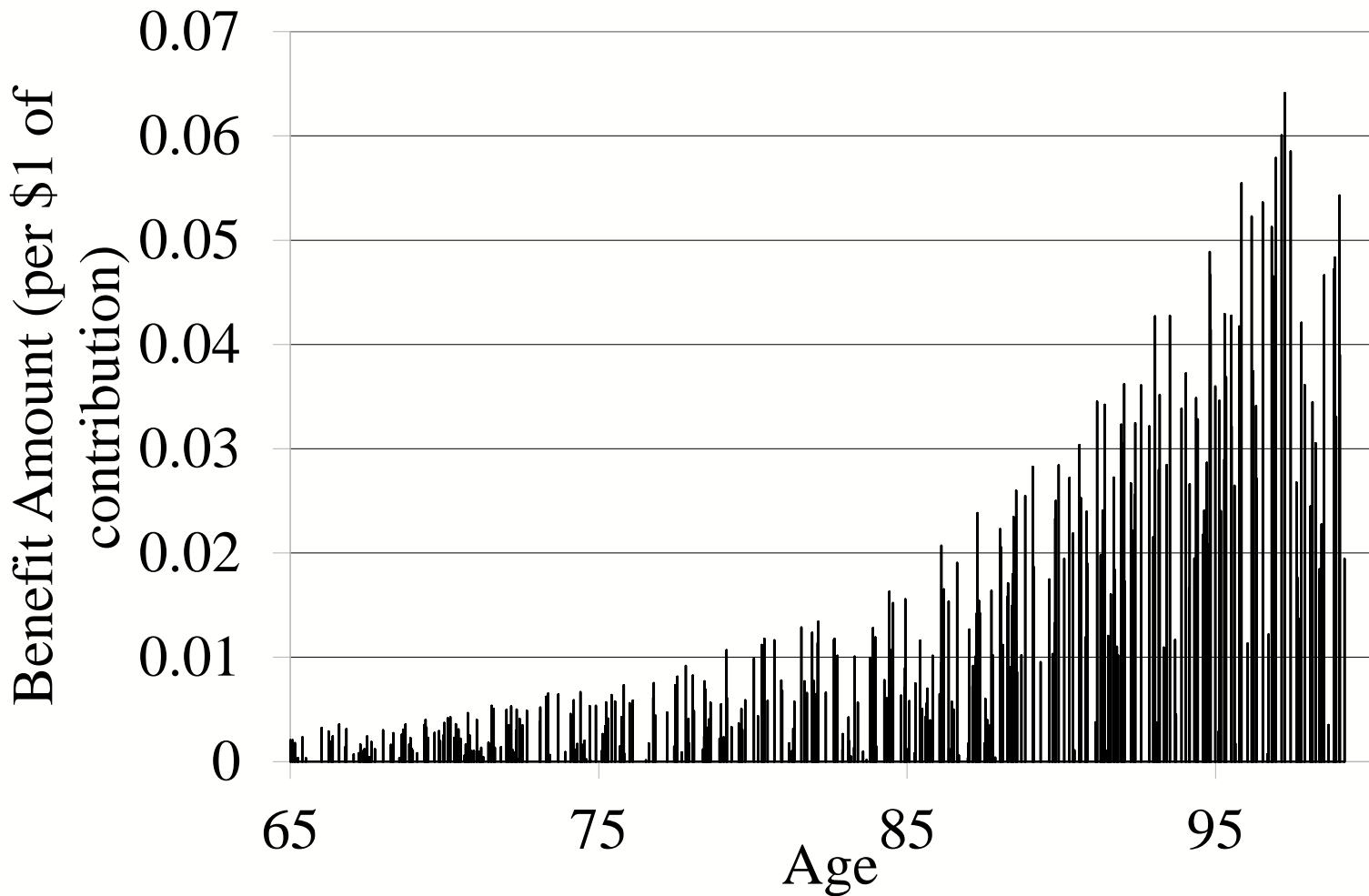


Example 5: A Simple Simulation

- A tontine fund with 220 members who
 - joined at age 65;
 - equiprobability male or female;
 - each member's contribution was a randomly selected amount between \$100 and \$100,000
- The next figure shows the mortality gains that a typical long-lived male could expect



Ex. 5: Payouts to a Long-living Male in a Small Tontine Fund





Ex. 5 Illustrates Two Problems

- **Tontine funds are noisy**
- **Tontine funds are backloaded**



1. We Can Reduce Noisiness with Monthly Payouts & More Participants

- We can reduce noisiness by
 - Making **monthly distributions**
 - rather than paying them at the time of each member's death; &
 - Increasing the number of investors
- Ex. 6: Imagine a large tontine fund with
 - monthly distributions, and
 - 5000 members
 - varying investment levels



Example 6: Sample Monthly Tontine Fund Statement for a Living Member

<i>Date</i>	<i>Amount</i>	<i>Balance</i>	<i>Description</i>
03/31		250,000.00	
04/02	67.17	250,067.17	Proceeds from FTP
04/03	25.21	250,092.38	Proceeds from FTP
04/05	55.14	250,147.52	Proceeds from FTP
04/07	135.41	250,282.93	Proceeds from FTP
04/07	48.91	250,331.84	Proceeds from FTP
04/12	52.29	250,384.13	Proceeds from FTP
04/15	102.54	250,486.67	Proceeds from FTP
04/20	159.46	250,649.13	Proceeds from FTP
04/21	139.68	250,785.82	Proceeds from FTP
04/22	17.82	250,803.63	Proceeds from FTP
04/25	124.81	250,928.44	Proceeds from FTP
04/28	55.32	250,983.76	Proceeds from FTP
04/30	57.91	251,041.67	Proceeds from FTP
04/30	(1041.67)	250,000.00	Payout of FTP proceeds



Example 6: Sample Statement for a Member Who Died during the Month

<i>Date</i>	<i>Amount</i>	<i>Balance</i>	<i>Description</i>
03/31		250,000.00	
04/02	67.17	250,067.17	Proceeds from FTP
04/03	25.21	250,092.38	Proceeds from FTP
04/05	55.14	250,147.52	Proceeds from FTP
04/07	135.41	250,282.93	Proceeds from FTP
04/07	48.91	250,331.84	Proceeds from FTP
04/12	(250,331.84)	0	Forfeited to FTP



2. We Can Eliminate Backloading with a Self-payback Mechanism

- Retirees want level payments
 - Not backloaded payments
- Each month, reduce a living member's account balance by repaying a portion of her initial contribution
- These “*self paybacks*” lead to
 - Level-payment “**Tontine Annuities**”; or
 - Inflation-adjusted Tontine Annuities



Ex. 7: Sample Tontine Annuity Statement for a Living Member with Self Payback

<i>Date</i>	<i>Amount</i>	<i>Balance</i>	<i>Description</i>
03/31		250,000.00	
04/02	67.17	250,067.17	Proceeds from FTP
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04/28	55.32	250,983.76	Proceeds from FTP
04/30	57.91	251,041.67	Proceeds from FTP
04/30	(2133.00)	248,908.67	Tontine-annuity Distribution



Here's the Self-payback Math

- Monthly payout equals the account balance divided by a monthly annuity factor
- E.g., the payout for 1st month of retirement (age 65):

$$\$2133.00 = \$251,041.67 \div 117.6939$$



Ex. 8: Sample Tontine Annuity Statement for a Living Member with Interest & Self Payback

<i>Date</i>	<i>Amount</i>	<i>Balance</i>	<i>Description</i>
03/31		250,000.00	
04/02	67.17	250,067.17	Proceeds from FTP
04/03	25.21	250,092.38	Proceeds from FTP
04/05	55.14	250,147.52	Proceeds from FTP
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04/25	124.81	250,928.44	Proceeds from FTP
04/28	55.32	250,983.76	Proceeds from FTP
04/30	57.91	251,041.67	Proceeds from FTP
04/30	1,000.00	252,041.67	Interest for the month
04/30	(2141.50)	249,900.17	Tontine-annuity Distribution



A Tontine Annuity Could Have Very Low Fees

- Approximates an actuarially-fair annuity
 - without insurance company profits and reserves
 - could be run by a discount broker
 - costs as low as 0.30% (30 basis points)
 - 0.10% expense ratio for an S&P 500 fund
 - + 0.20% for tontine recordkeeping



Adverse Selection is a Problem for All Annuities

- Underutilization is a problem for tontine annuities & traditional annuities
- Demand for annuities is low
 - The “annuity puzzle”
 - Adverse selection: those who buy annuities tend to live longer than those that do not
 - consequently, annuities are not priced very well for those with normal life expectancies
- Solution is broad coverage



Example 9: Tontine Pensions Would Provide Broad Coverage

- We modeled a 10%-of-pay “tontine pension”
- Employer hires 3600 employees each year
 - Workers start at 35; retire at 65
 - Starting pay = \$50,000; 4% annual pay increase
- 3% annual inflation
- 7% rate of return on investments
- Unisex life expectancy tables
- → 100,000 actives & 70,000 retirees

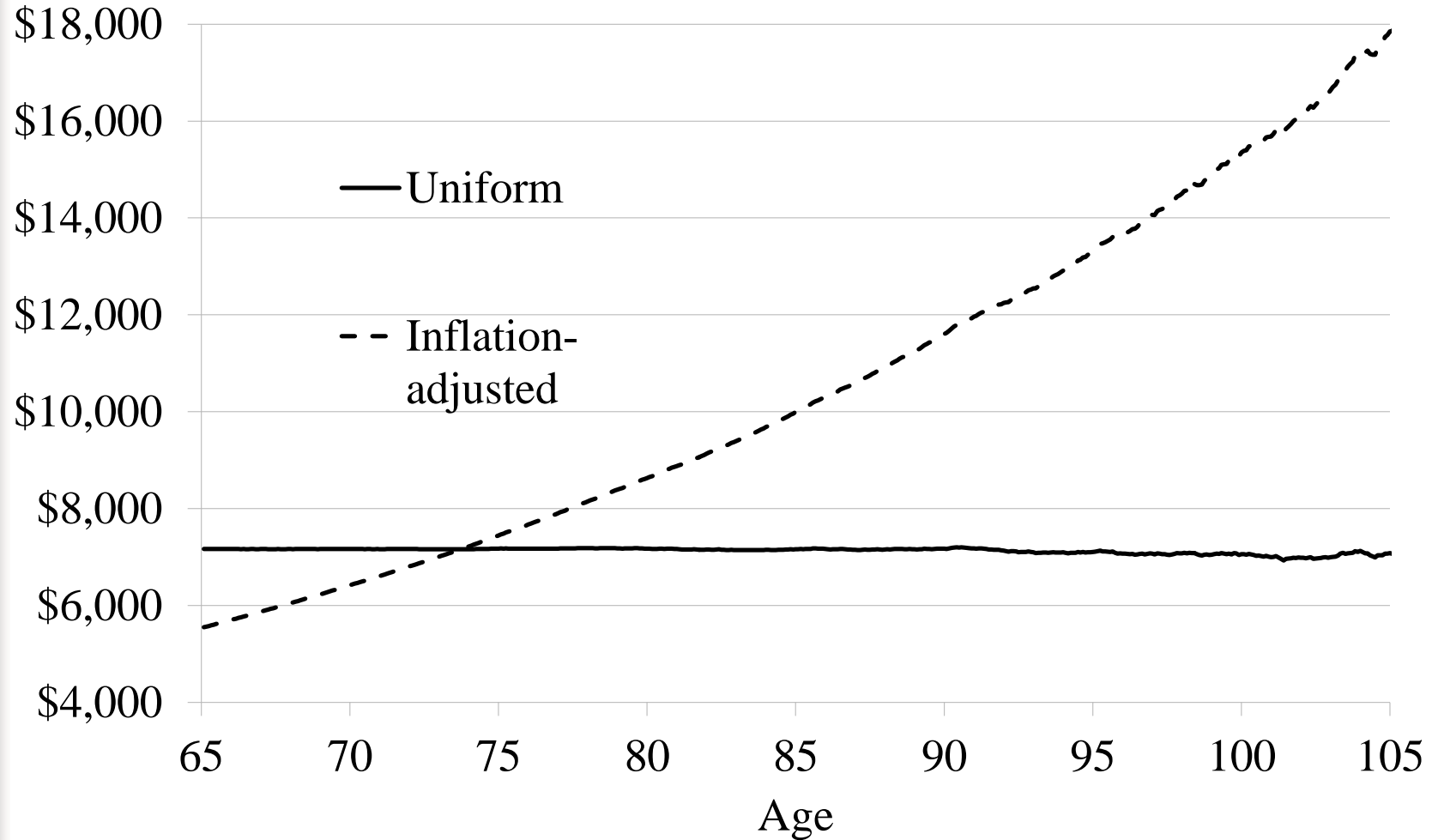


Example 9: Benefits from a 10%-of-pay Tontine Pension

- A worker who lives & works from 35-64 & retires at 65 would have:
 - Final salary = \$155,933
 - Starting retirement balance = \$843,376
- Uniform pension = \$7166/month
 - \$85,992/year; initial replacement ratio = 55.1%
- Inflation-adjusted pension starting at \$5549/month
 - ~ \$66,588/year; replacement ratio ~ 42.7%



Ex. 9: Monthly Payouts for a Typical Long-lived Member: Uniform vs. Inflation-adjusted





Example 10: Replacing CalSTRS

- CalSTRS traditional defined benefit plan
 - $B = 2\% \times \text{years of service} \times \text{final average pay}$
 - normal cost ~ 17% of compensation
 - Only 67% funded
 - unfunded liability ~ \$74 billion (another 15% of compensation)
- Partial solution: freeze the defined benefit plan & add a new tontine pension for future benefit accruals
 - Tontine pension will never be underfunded



Several Major Advantages of Tontine Pensions

- Unlike traditional pensions
 - Tontine pensions are always fully funded
 - Sponsors face no actuarial or investment risks
- Tontine pensions resemble actuarially-fair variable life annuities
 - Could be run by a low-fee discount broker
 - No insurance profits or reserves
 - Therefore, significantly higher benefits to retirees than with commercial annuities



More Advantages

- Tontines would be popular
 - E.g., a tontine for a team of firefighters will be perceived as fairer than a commercial annuity
 - With a commercial annuity, an early death seems to benefit the insurance company, but with a tontine pension, an early death benefits fellow firefighters
- Tontine pensions and annuities could be regulated & protected by fiduciary rules



Our Current Research

- **Individual Tontine Accounts**
- Tontines Investment Bonds
- Tontine Investment Funds

Individual Tontine Account

Individual Tontine Account (ITA)

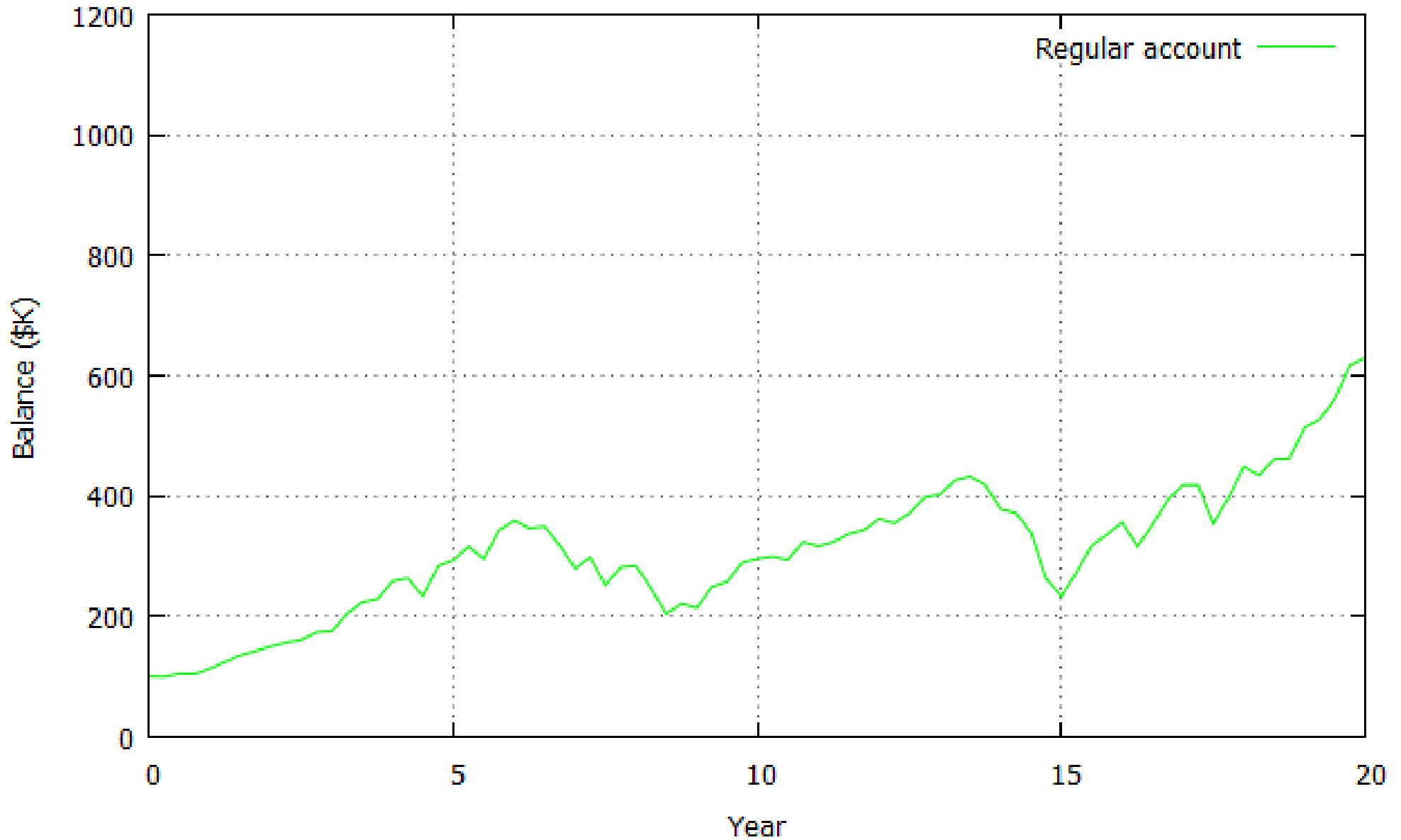
- Open and fund ITA account with broker (e.g., Ameritrade, Fidelity, Schwab, Vanguard, etc.)
- ITA contract:
 - 😊 I can invest in whatever I want
 - 😞 If I die, I forfeit account balance
 - 😊 When other ITA participants die, I get a piece of their account balances
 - 😞 I can only withdraw according to a predetermined schedule; but
 - 😊 I get to choose the withdrawal schedule

A Simple Example ITA

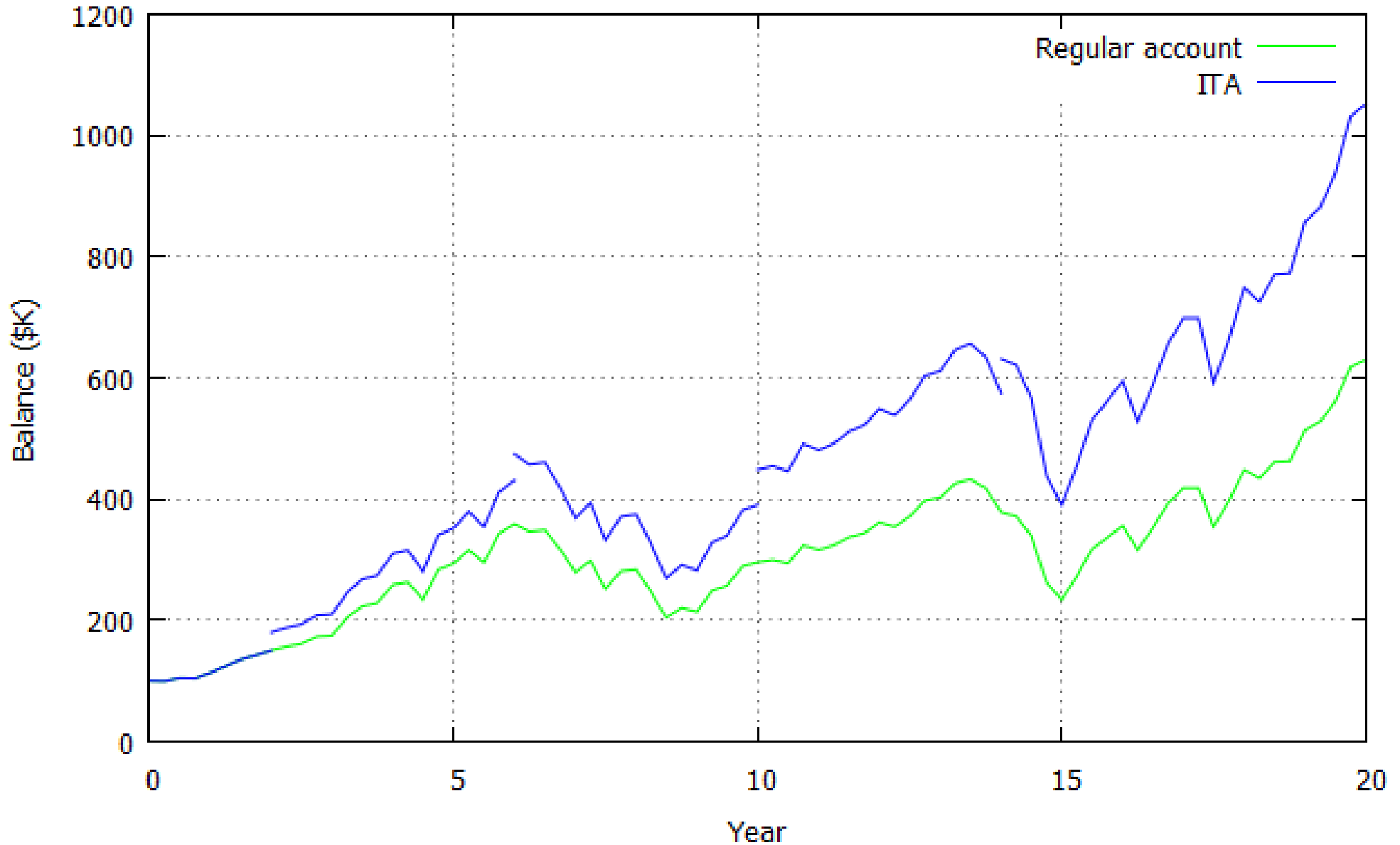
- I open and fund ITA with \$100K
- I contract to make a single withdrawal of entire balance 20 years later (if I survive)
- I invest entirely in S&P 500 (e.g. SPDRs)

Remark: this is what I select. Other ITA participants open at different times, fund with different amounts, choose different investments, and different withdrawal schedules.

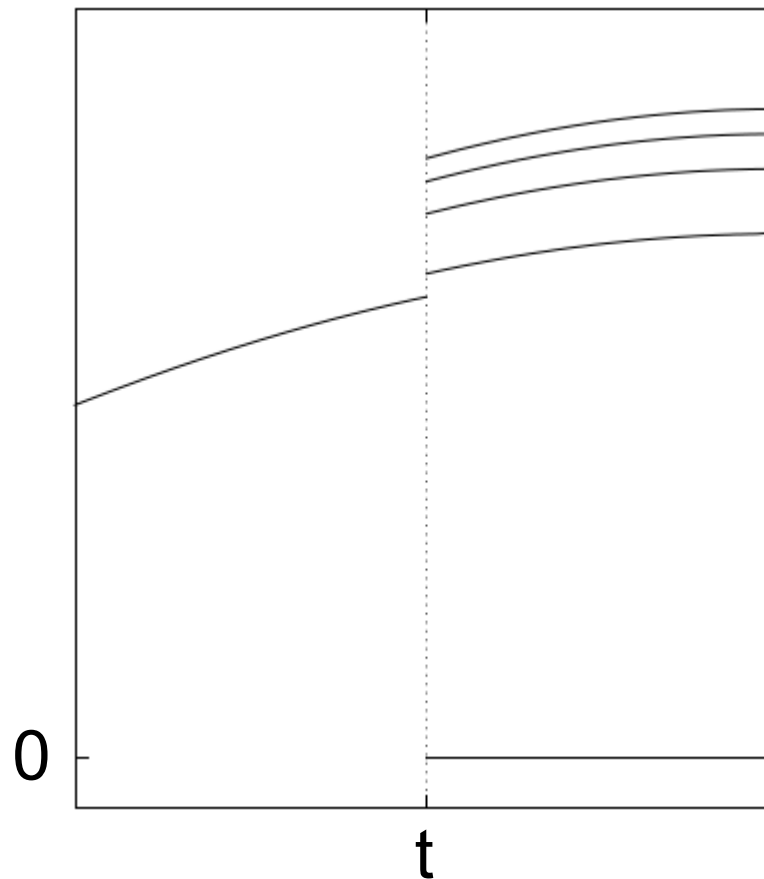
What Happens in a Regular Account?



What Happens in an ITA (if I survive)?



Details of a jump



- m possible outcomes (for m participants)
- If I die, my balance goes to 0: $S(t^+) = 0$
- If somebody else dies, my balance increases:
 $S(t^+) > S(t^-)$

Fact: my *expected* balance after jump equals balance before jump: $ES(t^+) = S(t^-)$

Fair Transfer Plan (FTP)

- If participant j dies:

- Balance of dying participant j :

$$S_j(t^+) = S_j(t^-) - S_j(t^-) = 0$$

- Balance of surviving participant i :

$$S_i(t^+) = S_i(t^-) + w_i / (1 - w_j) * S_j(t^-)$$

- The magic of the FTP is that w 's are picked such that $ES_k(t^+) = S_k(t^-)$ for every participant k

Calculating the FTP (the w's)

Need to solve for w_1, w_2, \dots, w_m :

$$\sum_{j \neq i} \frac{w_i}{1 - w_j} p_j s_j = p_i s_i \text{ for } i = 1, 2, \dots, m$$

Initialize $\theta_i = p_i s_i / (p_1 s_1 + p_2 s_2 + \dots + p_m s_m)$, $l = \theta_1$, $h = 2\theta_1$.

do

$$w_1 \leftarrow (l + h) / 2$$

for $i = 2, \dots, m$ **do**

$$w_i \leftarrow \frac{1}{2} - \frac{1}{2} \left[1 - \frac{4\theta_i}{\theta_1} w_1 (1 - w_1) \right]^{1/2}$$

done

if $\sum_{i=1}^m w_i < 1$ **then** $l \leftarrow w_1$ **else** $h \leftarrow w_1$

while $(h - l) / l > \varepsilon$

ITA vs. Regular Account

- Regular account
 - Start with \$100K
 - End with \$630K
($\approx 10\%$ annual return)
 - I get this whether I live or die
- ITA account
 - Start with \$100K
 - If I die, end with \$0
 - If I live, end with more than \$630K
 - End with *expected value* of \$630K

How Much More in ITA?

- Say my probability of dying during the 20 year is 40%.
- In regular account, I get \$630K whether I live or die
- In ITA, I get \$0 if I die. If I live, I get a random amount with expected value

$$\$630K * 1 / (1 - 0.40) = \$1,050K$$

- My expected value does not depend on who else is participating (large or small pool, old or young, rich or poor)
- But a big pool is better because my actual value will be close to its expected value (law of large numbers)

Recap

- A fair tontine is a flexible building block
- Use it to build a pension system, or
- Use it to build an individual tontine arrangement
- Magic step is the fair transfer plan

Historical Quote

“It is very difficult to establish [tontines] on sound principles, or according to the rules deduced from the theory of probabilities. ... To establish a fair tontine, it would be indispensable to class together none but individuals of the same age. But it would be impossible to establish any extensive tontine upon such principles, that is, on principles that would render the chances of the subscribers equal, and fully worth the sum paid for them.”


J.R. McCulloch. *A Treatise on the Principles and Practical Influence of Taxation and the Funding System*. Eyre and Spottiswoode, Her Majesty's Printers. Edinburgh, 1863.

- Problem solved, using fair transfer plan



Our Current Research

- Individual Tontine Accounts
- **Tontines Investment Bonds**
- Tontine Investment Funds



Example A: A Simple Zero Coupon Bond

- Imagine 1000 65-year-old retirees
 - Each contributes \$1000 to a fund that buys a \$1,000,000 **10-year zero coupon bond** with a 5% yield to maturity (5% interest)
- At maturity, bond would pay \$1,628,895
 - Which would be split equally among the 1000 original investors or their heirs
 - Each gets ~ \$1629.



Ex. B: A Simple Tontine Investment Bond

- Instead divide the \$1,628,895 but only among the **survivors**
- Suppose that only **800** survive to age 75
 - 800 winners (survivors) each gets ~ \$2036
 - **\$407 more!**
 - Translates to a **7.37% yield** to maturity
 - 47% higher yield to maturity
 - 200 losers (decedents) each loses \$1000
 - But for obvious reasons, they don't care!



Would You Take That 10-year Bet?

- People rarely buy lifetime annuities
 - Instead people invest in stocks and bonds
 - E.g., create a “laddered bond” portfolio
 - Buy 10 bonds worth \$10,000 each, laddered so that one bond matured in each of the next 10 years
- With a tontine investment bond, an investor could take on mortality risk but only over the relatively short term of the bond
 - i.e., for 10 years
 - vs. for life (with a lifetime annuity)



Our Current Research

- Individual Tontine Accounts
- Tontines Investment Bonds
- **Tontine Investment Funds**



Tontine Investment Funds

- Could work with any kind of investment
- Could invest in a family of mutual funds
 - E.g., College Retirement Equities Fund (CREF) operates 8 investment funds including stocks, bonds, money market & social choice
 - You could move among those funds, but you could not withdraw anything for 10 years
 - And you lose it all if you die before 10 years
- **Again, would you take that bet?**



How Would Tontine Investment Funds Work in the Real World?

- How would variations in the rate of return on the underlying investments affect the expected rate of return?
- How would the actual mortality experience of investors affect their rate of return?
- Can the fund be fair to investors with different ages, genders & investment levels?
- What are the technical problems?



About the Authors

- See Jonathan Barry Forman & Michael J. Sabin, *Tontine Pensions*, 163(3) UNIVERSITY OF PENNSYLVANIA LAW REVIEW 755 (Feb. 2015), <http://www.pennlawreview.com/print/index.php?id=468>.
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